Enhanced Assessment of Robustness for an Aircraft's Sliding Mode Controller

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The design of a multivariable sliding mode controller based on the equations of motion of an unmanned aircraft or remotely piloted vehicle (RPV) is described and an emphasis on robust eigenstructure assignment is given. The linear and nonlinear model responses are compared when subjected to the same variable-structure control design. The ability of the controller to cope with uncertain parameter variations is investigated as a problem of robustness assessment. It is shown that after sliding has commenced, the response of the objective system is tracked by the corresponding nonlinear or perturbed system. A measure of sensitivity is defined as the proximity of the actual system modal structure and response to that of the designed objective. The nonlinear system with variable-structure controller is also shown to be robust in the sense that this objective response is always recovered after a disturbance or parameter variation.

Introduction

VARIABLE-Structure Control Systems (VSCS) have received considerable attention in recent years as a possible solution to the problem of controlling systems for which statistics on parameter variations, noise and other variables are not available. A VSCS is characterized by a state feedback control structure that is switched as the system state crosses certain discontinuity surfaces in the state-space. The central feature of this control structure is the sliding mode, where the system state is constrained to lie within a neighborhood of all the switching surfaces. The sliding mode technique for control system design gained acceptance as a result of early work by Utkin[1] and Itkis.[2] Amongst other results, this work has shown that by changing the control structure, a system of variable structure was produced which could possess new properties not present in any of the composite structures. For example, an asymptotically stable system may consist of two unstable systems. Thus, a variable-structure system may be constructed by switching between a combination of two subsystems, each with a fixed structure and each operating in a specified region of the state-space. One of these subsystems can actually be unstable.[1] Another important property of VSCS is that after the commencement of the sliding mode, the system is unaffected by parameter variations and disturbances occurring within a subspace of the full state-space. More recent work by Ryan and Corless[3] has shown that variable-structure control may be used to establish almost certain convergence to a neighborhood of the origin for a class of uncertain systems.[4] In recent years, generalized theory and design techniques have been developed for sliding mode controllers, and these have been applied in the areas of model-following, model-reference adaptive control, tracking, and observers.[5–10]

The aircraft is a complex, highly nonlinear system whose aerodynamic parameters change considerably during flight, resulting in variations in the dynamic performance over the flight envelope. The work of Calise and Kramer[11] demonstrated that a VSCS can be applied to the aircraft using an optimal control approach. In this paper, variable-structure systems design techniques using eigenstructure assignment are used to formulate a nonlinear control law for the lateral motion model of an unmanned aircraft. The control law is assessed in its ability to compensate for parameter variations and nonlinearities using a

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nonlinear simulation of the vehicle and a perturbation of the nominal state matrices corresponding to a change in aircraft bank attitude angle. The robustness of the system is investigated with respect to the definition of Frank,[12] which states that a system is called robust if the system property of interest remains in a bounded region in the face of a class of finite-bounded perturbations. Frank also defines a system to be sensitive to parameter variations if a sensitivity measure $\boldsymbol{S}'$ is unlike $\mathbf{0}$. If a measure of $\boldsymbol{S}'$ is small, the system is termed insensitive. These definitions will be applied to a VSCS and a robustness measure for a sliding mode controller determined.

Design of a Sliding Mode Controller

The design of a VSCS entails the choice of discontinuity surfaces, specification of the discontinuous control, and determination of the associated switching logic. The switching surfaces are usually fixed hyperplanes in state-space passing through the state-space origin, the intersection of which forms the sliding subspace. The objective of the design is to drive the state of the system from an arbitrary initial condition to the intersection of the switching surfaces. The motion of the system is then effectively constrained to lie within a certain subspace of the full state-space, and the system becomes equivalent to a system of lower order termed the *equivalent system*.[1,5] Once the system state is sliding, it must be maintained in a neighborhood of the intersection manifold.

The aim of the design is to regulate the state vector to the equilibrium with an invariant response. The equivalent system must therefore by asymptotically stable. Stable sliding motion is assured by determining a set of hyperplanes such that the sliding mode defined by their intersection gives a specified performance to the equivalent lower-order system. The choice of control must ensure that the desired sliding mode is reached and maintained. This is the second stage of the design process, termed the *reachability problem*.[1]

The design may be formulated by considering the linear time-invariant system defined by

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \qquad \boldsymbol{x} \in R^n, \qquad \boldsymbol{u} \in R^m \tag{1}$$

It is assumed that $r(B) = m$ and that (A,B) are a controllable pair. The switching surfaces are usually intersecting hyperplanes S_j passing through the state-space origin. An m input multivariable system has m such manifolds, and the switching boundary is defined by the subspace spanned by the intersection of these manifolds:

$$S_j = (\boldsymbol{x} : \boldsymbol{c}^j \boldsymbol{x} = 0) \qquad j = 1,\ldots,m \tag{2}$$

Assembling the rows $\boldsymbol{c}^j$ into a full rank $(m \times n)$ matrix C gives

$$C\boldsymbol{x} = 0 \tag{3}$$

This motion results in the system state repeatedly crossing and recrossing a switching manifold defined by eigenvector directions in the state-space. All motions in the neighborhood of this switching manifold must be directed toward the manifold itself. This requires that

$$S_i \dot{S}_i \leqslant 0 \qquad i = 1,\ldots,m \tag{4}$$

In the ideal case (i.e., with no system uncertainties) the discontinuous motion actually results in a response known as *sliding motion*, which is characterized by an invariant direction within the manifold intersection.

A similarity transformation is used to map the system pair (A,B) into a controllable form[4] with

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{5}$$

where B_2 is $(m \times m)$ and nonsingular. The transformation matrix can be orthogonal to simplify the numerical calculation (as $T^T = T^{-1}$ for this case), and hence the change of basis takes the form

$$\boldsymbol{z} = T\boldsymbol{x}$$

The transformed state equations then become

$$\dot{\boldsymbol{z}} = TAT^T\boldsymbol{z} + TB\boldsymbol{u}$$

$$\boldsymbol{S} = CT^T\boldsymbol{z} \tag{6}$$

with $\boldsymbol{z}$ now partitioned as

$$\boldsymbol{z} = [z_1, z_2] \qquad z_1 \in R^{n-m},\ z_2 \in R^m \tag{7}$$

By partitioning the matrices in Eq. (6) accordingly,

$$TAT^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{8}$$

and

$$CT^T = [C_1\ C_2]$$

The partitioned state equations then give

$$\dot{z}_1 = A_{11}z_1 + A_{12}z_2 \tag{9}$$

$$\dot{z}_2 = A_{21}z_1 + A_{22}z_2 + B_2\boldsymbol{u} \tag{10}$$

$$\boldsymbol{S} = C_1 z_1 + C_2 z_2 \tag{11}$$

It is assumed that CB is nonsingular, for in this case the sliding motion is independent of the actual control $\boldsymbol{u}$. Geometrically,

$$|CB| \neq 0 \Rightarrow N(C) \cap R(B) = \{0\} \tag{12}$$

Thus, when motion lies entirely within $N(C)$ during the sliding mode, the behavior of the ideal system is unaffected by the controls as they act within $R(B)$. Since the hyperplane matrix C is to be designed, Eq. (12) may be imposed. This assumption implies that C is also nonsingular since

$$|C_2||B_2| = |C_2 B_2| = |CT^T \cdot TB| = |CB| \neq 0 \tag{13}$$

Therefore, noting that during sliding the switching function $\boldsymbol{S}$ is identically equal to zero, the state lies simultaneously in each of the hyperplanes $S_j, j = 1,\ldots,m$. Equation (11) thus gives

$$z_2 = -C_2^{-1} C_1 z_1 \tag{14}$$

The ideal sliding mode is governed by Eqs. (9) and (14), an $(n - m)$th-order system where z_2 is the state feedback control. Closing the loop in Eq. (9) with the state feedback from Eq. (14),

$$\dot{z}_1 = (A_{11} - A_{12}C_2^{-1} C_1)z_1 \tag{15}$$

An asymptotically stable system must be ensured. The sliding surface design problem now becomes that of fixing $(C_2^{-1} C_1)$ to give $(n - m)$ left-hand half-plane eigenvalues of $(A_{11} - A_{12}C_2^{-1} C_1)$. The method chosen for the aircraft problem under consideration in this paper is eigenstructure assignment. The eigenvalues of the system determine the rates of decay of the modes while the eigenvectors determine the contribution of each mode to the various states. For an aircraft flight control design problem, the modal behavior that maintains the required handling qualities is well-known. During sliding,

$$C\boldsymbol{x} = \mathbf{0} \tag{16}$$

Differentiating Eq. (16) with respect to time,

$$C\dot{x} = 0 \tag{17}$$

Substituting from Eq. (1),

$$C(Ax + Bu) = 0 \tag{18}$$

i.e.,

$$u = u_{eq} = -(CB)^{-1}CAx$$

where u_{eq} is defined as the equivalent control that governs the response during sliding.[1,5] Substituting back into Eq. (1),

$$\dot{x} = A_{eq}x = (A - BK)x \tag{19}$$

where

$$K = (CB)^{-1}CA$$

Thus, the equivalent system matrix during sliding is A_{eq}. The n eigenvalues and associated eigenvectors of this matrix will determine the responses during sliding. Equation (16) expresses m of the states in terms of the remaining $(n - m)$, and hence a reduction in system order from n to $(n - m)$ is possible. It follows that m of the eigenvalues of A_{eq} will necessarily be zero-valued. Now,

$$C\dot{x} = 0 \Leftrightarrow C(A - BK)x = 0$$

$$\Leftrightarrow R(A - BK) \subseteq N(C) \tag{20}$$

Let the eigenvalues of $(A - BK)$ be λ_i, $i = 1,\dots,n$ with corresponding eigenvectors w_i. Then, from Eq. (20),

$$C(A - BK)w_i = \lambda_i C w_i = 0 \tag{21}$$

i.e.,

$$\lambda_i = 0 \quad \text{or} \quad w_i \in N(C)$$

Assume λ_i, $i = 1,\dots,n - m$ are distinct and nonzero. Then, by assigning the $(n - m)$ corresponding eigenvectors, the null space of C is fixed, although C is not unique:

$$CW = 0 \qquad W = [w_1,\dots,w_{n-m}]$$

and so

$$[C_1\ C_2]\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0 \qquad \text{where} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = TW \tag{22}$$

$$\Leftrightarrow C_2[C_2^{-1}C_1 \quad I_m] = 0 \tag{23}$$

or

$$(C_2^{-1}C_1)V_1 = -V_2$$

Therefore, provided V_1 is nonsingular, (C_2^{-1}) is uniquely determined; by choosing $C_2 = I_m$, it follows that

$$C = [C_1 \quad I_m]T \tag{24}$$

and the sliding surface design problem is solved.

The essential feature of a variable-structure control system (VSCS) is that the state feedback control structure utilized is switched as the system state crosses certain discontinuity surfaces in the state-space. The so-called *sliding mode* behavior is an idealized concept that might occur when $n - m$ of the directions of motion in the state-space become invariant (i.e., become linearly related to $n - m$ eigenvectors). If the interval between switching instants becomes infinitesimally small, then the $n - m$ subsystem becomes a free system, with direction in state-space given by the intersection of $n - m$ eigenvectors. In the case of parameter variations, the system can be described as having uncertain dynamics, and for this case, ideal sliding motion can only be approached asymptotically. However, the VSCS design provides a very powerful deterministic design approach for uncertain systems and has strong robustness properties. The VSCS can be thought of as a closed-loop system with an adaptively varying state-feedback gain. Thus, a feedback control law of the form

$$u = -K^T x \tag{25}$$

with

$$K^T = k^T + \Delta k^T$$

is used where the control matrix K^T is the sum of the fixed and switched matrices k^T and Δk^T. The linear controller is merely a state feedback controller while the nonlinear feedback controller incorporates the discontinuous elements of the control law. The control structure used in the examples that follow has the unit vector form with

$$u_{\text{TOTAL}} = u_L + u_{NL} \tag{26}$$

$$u_L = Lx \tag{27}$$

and

$$u_{NL} = \frac{Nx}{\|Mx\|} \tag{28}$$

The null spaces of N, M, and C are coincident, as described by Ryan and Corless.[3] The role of the linear control is to force the range space states to zero asymptotically to attain the sliding mode. To attain the null space of C in finite time, the nonlinear control component is required. This nonlinear component must be continuous whenever S is nonzero but is discontinuous (or zero) during sliding. The control system design is accomplished in the following way. Starting from the transformed state z given by Eq. (7), a second nonsingular transformation matrix is defined by

$$y = T_2 z \tag{29}$$

This is such that

$$y_1 = z_1 \tag{30}$$

$$y_2 = C_2^{-1}C_1 z_1 + z_2 \tag{31}$$

Comparing this to Eqs. (11) and (14), it is seen that the points of the state-space at which S equals zero are precisely those points at which y_2 is equal to zero. Thus, to obtain the sliding mode, it is necessary to force y_2 and $\dot{y}_2$ to become identically zero. In this way, the linear part of the control law, Eq. (27), is formulated. Similarly, the nonlinear control component is found by considering that whenever the system is sliding, i.e., y_2 is equal to zero, the control must be discontinuous, but for all other y_2, the control will be continuous. This calculation involves finding the positive-definite solution of a Lyapunov equation. For more details of this particular control system development see, e.g., Dorling and Zinober.[4]

A common criticism of switched designs is the "chatter" inherent in a discontinuous control structure. However, it has been demonstrated that switching need only be shown at the design stage to ensure sliding.[10] A smoothing control parameter δ can be used when the controller is applied. In this case, Eqs. (26–28) become

$$u_{\text{TOTAL}} = Lx + \frac{Nx}{\|Mx\| + \delta} \tag{32}$$

The effect of δ is to provide a compromise between the true sliding condition and motion in close proximity to the hyperplane intersections.

Robustness Assessment of the Sliding Mode Controller

Robustness and sensitivity are both concerned with the ability of a system to tolerate uncertainties or nonlinearities. The following definitions attributable to Frank[12] are given:

1) *Parameter Sensitivity*. A system is said to be sensitive to parameter variations if a sensitivity measure S' is unlike zero. In the special case $S' = 0$, the system is said to be zero sensitive. If a measure of S' is small, the system is termed insensitive.

2) *Robustness*. A system is called robust if the system property of interest remains in a bounded region in the face of a class of finite-bounded perturbations.

For the assessment of a sliding mode controller, the "bounded region" is taken to be the manifold intersection in state-space corresponding to the null space of C, and a measure of robustness may be taken to be the "distance" of the state vector from the desired null-space objective at any time t. Parameter variations, and thus the sensitivity of the system, may be determined by looking at the variation of this distance with time.

When the state begins to slide down the intersection of the switching surfaces, the system will exhibit the same dynamics as a system given by

$$\dot{x} = (A + BL)x \tag{33}$$

with L as defined by Eq. (27). Thus, sliding may be demonstrated by running a full simulation, storing the state vector when sliding has commenced, and using this as the initial condition for the system of Eq. (33). This is necessary as the system response is "piecewise" linear only between switching instants. The responses of the full simulation and Eq. (33) should match for $t > t_s$ where t_s is the time at which sliding begins.

To establish when sliding begins, one may look either at the nonlinear control component to see when this becomes discontinuous or at the magnitude of S. A more sophisticated method of determining whether or not a system is essentially following the specified dynamics is to examine the orthogonal projection of the state vector into the null space of C at each simulation sample point. The orthogonal projection into the null space of C is achieved using the Singular Value Decomposition method.[21] Let

$$C = [U_r \; \bar{U}_r]\Sigma[V_r \; \bar{V}_r]^T \tag{34}$$

where Σ is the diagonal matrix of singular values of C. If $r = \text{rank}(C)$, then $U_r \in R^{m \times r}$, $\bar{U}_r \in R^{m \times (n-r)}$, $V_r \in R^{n \times r}$, and $\bar{V}_r \in R^{n \times (n-r)}$. The projection matrix P is formed as

$$P = V_r V_r^T \tag{35}$$

The state vector x is then projected into the null space of C using

$$x_p = Px \tag{36}$$

The norm of x_p then gives the "distance" of the point in state-space from the switching boundary. From the definition given by Frank,[12] this is taken to be the measure of robustness, as if the norm of the projection vector is small, the dynamics of the system must be close to those of the null space of C. The norm thus provides a measure of the eigenstructure robustness.

By testing the ratio of the actual value of each component of the state vector to the equivalent component after projection, information about the position of each state relative to the null space may be determined. This ratio should be seen to converge to unity for all n states; the system is then sliding. The norm used here is the 2-norm (or Euclidean norm) defined by

$$\|x\| = (x_1^2 + \cdots + x_n^2)^{\frac{1}{2}} \tag{37}$$

The projection idea developed here also provides a method for choosing a parameter δ. One may look at the norm of the orthogonal projection into the null space of C for the nominal system with δ zero-valued and compare this with the norm for a series of small positive values of δ. As expected from the control form, the responses will converge to slightly differing values. One requires δ to be sufficiently large to eliminate the "chattering" behavior of the nonlinear control, but small enough such that the state of the system may be as close as possible to the prescribed null space. Simulation studies incorporating the projection idea may be used to analyze this.

Aircraft Dynamic Model

The equations of motion of the fixed-wing aircraft are normally decoupled into two subsystems, each describing the longitudinal and the lateral motions. The longitudinal motions are concerned with forward velocity and pitching excursions, whilst the lateral motions relate to roll, yaw, and sideslip velocity. In this work, a controller is designed for the lateral motion of a particular remotely piloted vehicle for which the equations of motion and the aerodynamic parameters are well defined and published elsewhere.[13] The lateral and longitudinal motions are assumed decoupled as far as the control system design is concerned. A "stick-fixed" linearization of the nonlinear system gives the following linear model:

$$\dot{x} = Ax + Bu \tag{38}$$

where

$$x = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \\ \zeta \\ \xi \end{bmatrix} \begin{matrix} = \text{sideslip velocity (m s}^{-1}) \\ = \text{roll rate (rad s}^{-1}) \\ = \text{yaw rate (rad s}^{-1}) \\ = \text{roll angle (rad)} \\ = \text{yaw angle (rad)} \\ = \text{rudder angle (rad)} \\ = \text{aileron angle (rad)} \end{matrix} \tag{39}$$

$$u = \begin{bmatrix} \zeta_c \\ \xi_c \end{bmatrix} \begin{matrix} = \text{rudder angle demand (rad)} \\ = \text{aileron angle demand (rad)} \end{matrix} \tag{40}$$

The unmanned aircraft under consideration here has trim flight linear lateral motion model matrices corresponding to a 33 m s^{-1} airspeed, as shown in the Appendix.

The lateral aircraft model is characterized by three modes: the roll, the Dutch roll, and the spiral mode:

$$\begin{matrix} -0.502 \pm j3.508 & \text{Dutch roll} \\ -8.360 & \text{roll} \\ 0.122 & \text{spiral} \\ 0.0 & \text{yaw attitude mode} \\ -10.0 & \text{rudder mode} \\ -5.0 & \text{aileron mode} \end{matrix} \tag{41}$$

It should be noted that the open-loop model has a spiral divergence mode at $s = +0.122$. The model is also strictly nonminimum-phase with a positive zero at $+12.61$ occurring in the roll subsystem as well as the one positive real open-loop pole. This has obvious implications for the stability margin as the possibility always exists of achieving a nonminimum-phase closed-loop design even when using all the degrees of freedom available in full-state feedback.

As well as tailoring the stability margins and time response of the closed-loop system, it is necessary to consider the weighting of the various aircraft modes. For example, it is necessary to decouple roll from yaw and Dutch roll from both roll and aileron control actuation. These motions are normally coupled via the airframe, with roll giving rise to sideslip and vice versa. This essentially amounts to assigning a prescribed eigenstructure for the closed-loop system. In this way, the control system design can be tailored to satisfy MIL-8785C specifications on damping, frequency response, and modal decoupling.[14] The RPV in question was built to develop and test control design techniques for piloted aircraft. The flying qualities specification is thus used to illustrate the design procedure in general and not for an unmanned aircraft in particular. The MIL-8785C specification is a combat aircraft standard. Whilst not strictly appropriate to the unmanned aircraft, the closed-loop assignment provides a useful cross comparison with the work of Sobel et al.[15,16] and Alag and Duke,[17] with the additional robustness properties exhibited by the sliding mode design approach.

Aircraft Sliding Mode Design

The development of a sliding mode controller for the lateral motion aircraft example requires the selection of an eigenstructure for the sliding or null space part of the control. The lateral motion modes are required to be such that the spiral mode has a very slow divergence, the roll mode is fast and stable, and the Dutch roll mode has a 0.7 damping ratio. A set of $(n-m)$ null-space eigenvalues are chosen according to MIL-8785C specification[14] as

$$\begin{aligned} \lambda_1,\lambda_2 &= -2.0 \pm j1.0 && \text{(roll mode)} \\ \lambda_3,\lambda_4 &= -1.5 \pm j1.5 && \text{(Dutch roll mode)} \\ \lambda_5 &= -0.05 && \text{(spiral mode)} \end{aligned} \tag{42}$$

During sliding, these correspond to the $(n-m)$ nonzero roots of the characteristic polynomial of A_{eq}, where A_{eq} is given by Eq. (19). In the idealized sliding case, these roots are also the poles that cancel $(n-m)$ transmission zeros of the closed-loop system. Hence also, in the idealized case, the $n-m$ null-space modes become unobservable in the measure $S = Cx$. The *zero observability* is equivalent to *zero sensitivity* as defined by Tomovic.[18] The $n-m$ null-space modes are therefore *zero sensitive* in S during sliding action.

To ensure that the various aircraft modes appear in the correct weighting in particular motions, the desired eigenvectors are

$$v_1,v_2 = \begin{bmatrix} 0 & 0 \\ 1 & * \\ 0 & 0 \\ * & 1 \\ * & * \\ * & * \\ * & * \end{bmatrix}, \quad v_3,v_4 = \begin{bmatrix} 1 & * \\ 0 & 0 \\ * & 1 \\ 0 & 0 \\ * & * \\ * & * \\ * & * \end{bmatrix}, \quad v_5 = \begin{bmatrix} 0 \\ * \\ * \\ 1 \\ * \\ * \\ * \end{bmatrix} \tag{43}$$

where * denotes that the magnitude of the element is unimportant.[19] The roll mode should thus show up dominantly on roll rate but not on sideslip velocity or yaw rate; banking maneuvers should not incur sideslip buildup. Since $\phi = \int p \, \mathrm{d}t$, some mode content must be expected on roll angle and also on the aileron and rudder states if the surfaces are controlling the mode. Similar arguments apply to the Dutch roll mode. There must be no coupling of Dutch roll into either roll rate or roll angle to satisfy the handling quality criteria. The spiral mode should not appear on sideslip velocity in order to avoid sideslip in steady turns. These desirable eigenvectors are projected in a linear least-squares sense into the allowable subspace.[20] The normalized assignable eigenvectors are found to be

v_1	v_2	v_3	v_4	v_5
0.006087	−0.001114	0.030086	0.997113	0.000000
0.297003	−0.833108	0.000000	0.000000	−0.008176
−0.084225	0.078102	0.044930	0.030086	0.049262
−0.285423	0.273843	0.000000	0.000000	−0.985232
0.049310	−0.014396	−0.004948	−0.025005	−0.985232
−0.003615	0.020033	0.009989	0.033968	−0.003057
−0.105730	0.183848	0.005774	0.000343	0.008812

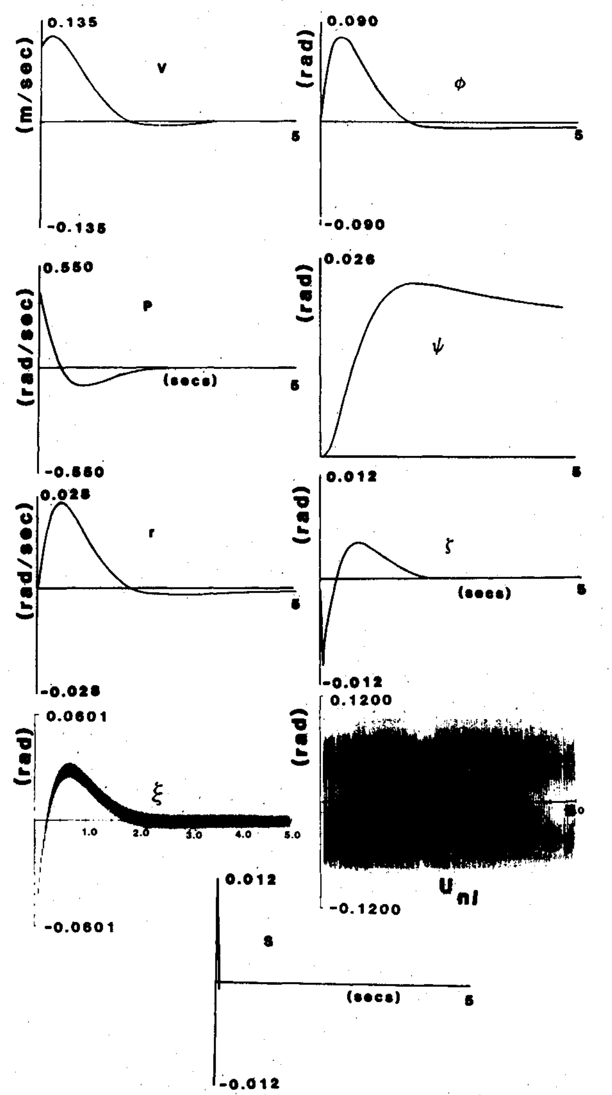

Fig. 1 **Lateral motion responses for the nominal linear system.**

The C matrix providing the null-space dynamics is

$$C = \begin{bmatrix} -0.0028 & 0.1493 & -0.1344 & -0.2240 & -0.0362 & 0.0 & 1.0 \\ -0.0279 & 0.0309 & -0.2037 & 0.0782 & -0.0006 & 1.0 & 0.0 \end{bmatrix}$$

The equivalent system matrix A_{eq} is identical to the system matrix A in its first five rows, with the last two rows being

$$\begin{matrix} 0.0698 & 0.1787 & -1.1648 & 0.2740 & 0.0000 & -2.0844 & 0.8866 \\ 0.0650 & 1.4234 & -0.5188 & -0.0291 & 0.0000 & -1.2527 & 4.2754 \end{matrix}$$

The eigenvalues of A_{eq} are

$$\begin{matrix} -1.5 \pm j1.5 \\ -2.0 \pm j1.0 \\ -0.05 \\ 0.0 \\ 0.0 \end{matrix}$$

The nonlinear controller design of Eqs. (26–28) then follows, with the discontinuous control selected according to actuator limitations.

Figure 1 shows the state responses for a linear model of the aircraft at 33 m s^{-1} airspeed with initial conditions of 0.1 m s^{-1} in v and 0.5 rad s^{-1} in p. One nonlinear control component and plot of S are illustrated. From the switching on the

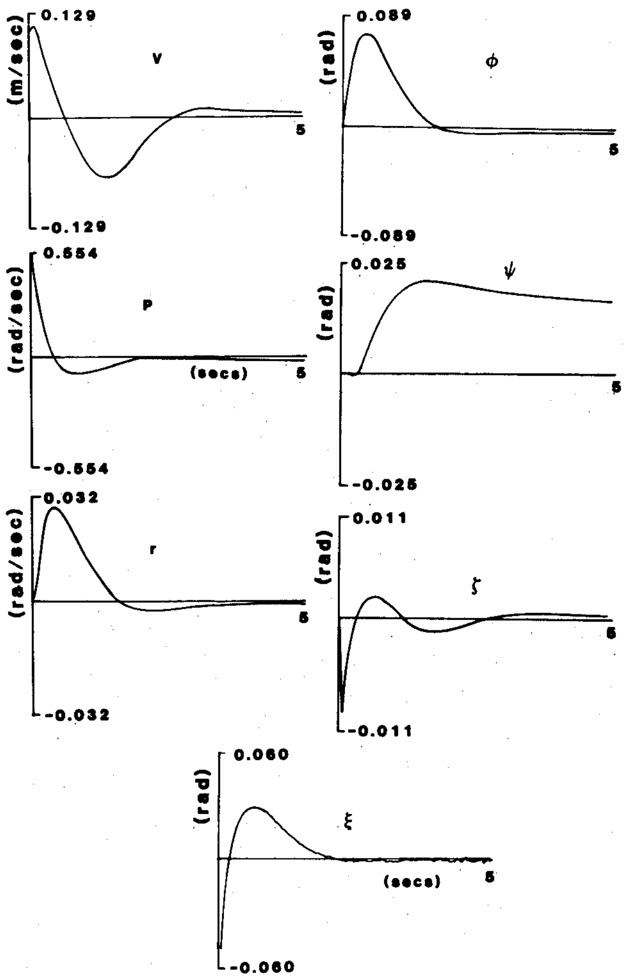

Fig. 2 Lateral motion state responses for the full nonlinear simulation.

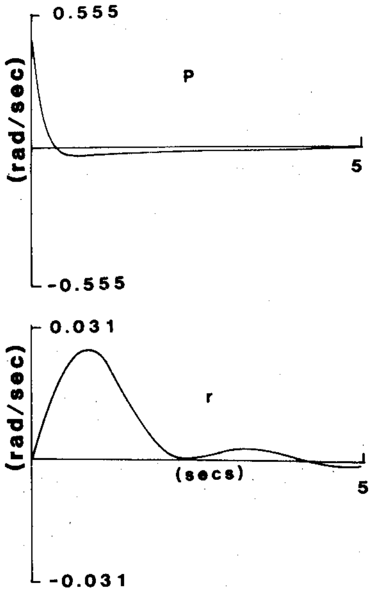

Fig. 4a State responses of the nonlinear simulation controlled by the linear component of the VSS controller only.

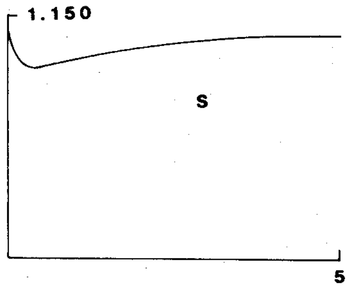

Fig. 4b Component of *S* for this simulation.

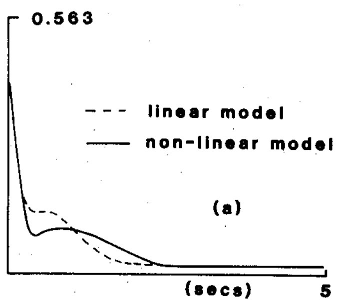

Fig. 3a Norm variation.

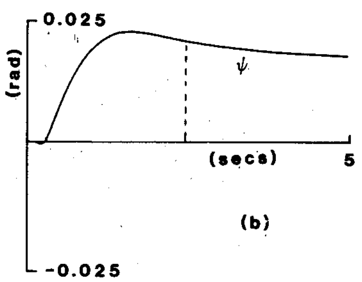

Fig. 3b Nonlinear system response tracking desired objective after sliding.

nonlinear control component, which begins very quickly, and the small value of S, it can be seen that the desired sliding regime is reached almost immediately. The roll mode, which is associated dominantly with the roll rate p and thus also with the bank angle ϕ, has the desired fast, first-order response. The Dutch roll mode appears only in the sideslip v and yaw rate r responses and has the required damping. The spiral mode appears on the bank angle state ϕ. The actuator demands are acceptable, the "chatter" being due to the discontinuous nature of the nonlinear control component given by Eq. (28). The discontinuities can occur at each simulation sample point because a sampled-data approach is applied to an otherwise continuous time system design.

A nonlinear simulation of the unmanned aircraft has been used to provide a realistic assessment of the performance of the VSS controller in the presence of parameter variations. The simulation incorporates both the full force and moment lateral and longitudinal dynamics together with cross-coupling effects. Figure 2 shows the responses of the lateral motion states. An initial comparison of the sliding mode controller's performance using the linear and nonlinear system models shows that the responses are broadly similar. Figure 3a shows the variation of the 2-norm of the orthogonal projection of the state vector into the null space of C at every sample instant for both the linear and nonlinear models. The null space is achieved at approximately 2.5 s by both systems. The response of the system of Eq. (33) is shown tracking the nonlinear model response for one of the system states after this time in Fig. 3b.

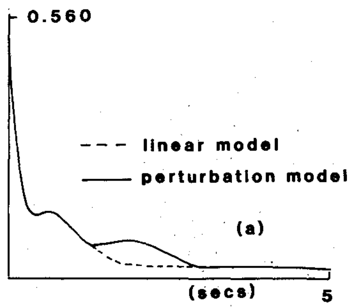

Fig. 5a Norm variation.

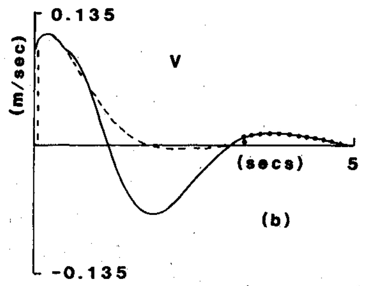

Fig. 5b Perturbed system response tracking desired objective after sliding.

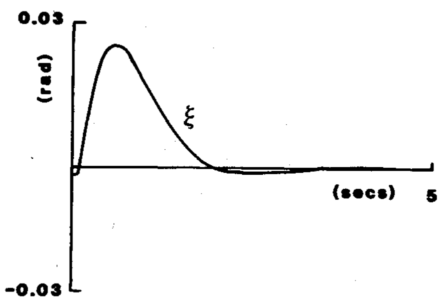

Fig. 6a Smoothed aileron response ($\delta = 0.1$).

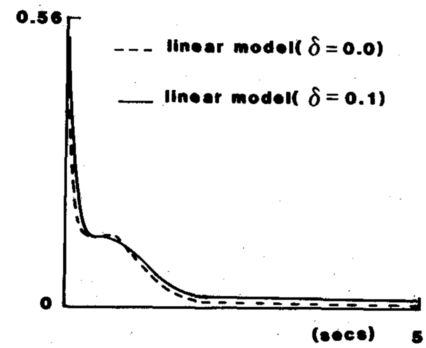

Fig. 6b Norm variation.

In order to demonstrate the lack of robustness resulting from the use of simple linear state feedback control, a second nonlinear simulation of the full-force nonlinear aerodynamic equations was run. The control utilized here was purely the linear state feedback defined by $u_L = Lx$, i.e., the VSS controller minus the switched component. Figure 4a shows the responses of the roll rate p and yaw rate r for this simulation. These should be compared with the equivalent responses in Fig. 2a. The results show clearly that the modal objectives expressed in Eq. (42) and (43) are not maintained when using this linear feedback alone applied to the nonlinear aircraft model. This illustrates that the linear feedback design does not maintain the correct eigenstructure during parameter variations and is therefore not robust in this sense. Figure 4b shows a plot of S_1 (the first component of S), and it can be seen that this is not close to zero. This further demonstrates the lack of eigenstructure robustness in the sense defined in this paper. The definition of robustness given here corresponds to the eigenstructure of the null space of C as defined by Eq. (14).

To assess further the nonlinear controller derived here, a second linear simulation was carried out. The system matrix at time $t = 0$ was the nominal A matrix used to develop the controller for which the bank angle $\phi = 0$. At $t = 0.5$ s, this nominal A matrix was perturbed to represent a change in bank angle to $\phi = 0.15$ rad. Figure 5a shows the norm of the orthogonal projection of the state vector for two simulations, one of the nominal linear system alone and one of the perturbed system described. Figure 5b shows the responses of the sideslip velocity. The continuous line shows the state response for the perturbed system. At $t = 0.1$ s, the $(A + BL)$ response is initialized with the system state vector from the nominal A part of the simulation. This dashed line shows that the nominal system is sliding at $t = 0.1$ s. At $t = 0.5$ s, the null-space response and perturbed A response are seen to diverge due to the parameter changes. However, it is seen from the dotted line that by $t = 3.5$ s, the null-space response initialized here once again tracks the system response. It should be stressed that the null-space response from Eq. (33) calculated using the nominal A matrix is used throughout this test. It has thus been shown that the nonlinear controller developed is robust in the sense that it will compensate for parameter changes, giving the designed system response. The robustness property previously defined is demonstrated as S is forced to zero after a parameter change.

The null-space eigenstructure is thus maintained close to the required objective, and so the required handling qualities are preserved.

The effect of the smoothing parameter δ has also been studied. From simulation studies, a value $\delta = 0.1$ removes the switching for both linear and nonlinear cases. Figure 6a shows the smoothed aileron response, which should be compared with the equivalent response in Fig. 1. The true sliding condition may no longer be attained. The projection idea may now be used to assess this new smoothed motion, which is desired to be in close proximity to the hyperplane intersections. Figure 6b shows the norm of the orthogonal projection for the nominal linear system, with $\delta = 0$ against that for the linear system with $\delta = 0.1$. As is to be expected, the two responses converge to slightly differing values. However, they are seen to be close enough such as to be a very good approximation to the desired dynamics.

Conclusion

The design and assessment of a sliding mode controller for the lateral motion subsystem of an unmanned aircraft has been considered. Earlier work has demonstrated that variable-structure control theory can be used in the design of flight control systems and showed that the resulting response (in the absence of uncertainties) is almost identical to that obtained using linear control theory. The sliding surface design problem in this paper has been tackled by an eigenstructure asssignment approach, unlike the alternative optimal control methods used previously. It has been shown that the system behavior in the sliding mode is prescribed directly to give the desired handling qualities using this approach. Furthermore, the robustness assessment of the variable-structure controller generated has been extended, and it has been shown that nonlinear and perturbed systems can be forced to exhibit the behavior described by the null-space dynamics. Thus, a more extensive analysis for VSCS is now possible, providing a greater insight into and measure of the robustness properties than can be gained by system responses alone.

Finally, it has been shown that practical implementation of VSCS is now possible using a smoothed form of control law, which will not exceed actuator rate limits but will still enable a close approximation to the desired dynamics to be attained in a reasonable time.

Appendix

The aircraft model contains the full-force aerodynamic motions in all axes and includes lift stall characteristics. The longitudinal-lateral coupling has been retained in the simulation together with the full aerodynamic nonlinearities. The linearization was established by considering small changes about a chosen state trajectory, omitting coupling terms from the design:

$$A = \begin{bmatrix} -0.277 & 0.000 & -32.900 & 9.810 & 0.000 & -5.432 & 0.000 \\ -0.103 & -8.325 & 3.750 & 0.000 & 0.000 & 0.000 & -28.640 \\ 0.365 & 0.000 & -0.639 & 0.000 & 0.000 & -9.490 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -10.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -5.000 \end{bmatrix} \qquad B = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.000 \\ 0.000 & 0.000 \\ 0.000 & 0.000 \\ 0.000 & 0.000 \\ 20.000 & 0.000 \\ 0.000 & 10.000 \end{bmatrix}$$

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